Indian Statistical Institute Bangalore
Back Paper Examination
Date: June 11, 2019.

Topology   B.Math II   Instructor: Santhosh Kumar P   Total : 50

All questions carry equal marks.

(1) Define separable topological space. Show that if X has a countable basis, then X is separable. Is the converse true ? Justify.

(2) (a) Give an example of connected space which is not path connected.
  (b) Let A be a subset of a Hausdorff space X. Show that if x is a limit point of A if and only if every nbd of x contains infinitely many points of A.

(3) Show that
  (a) If X is a non-compact metric space, then there exists an unbounded continuous function $f \colon X \to \mathbb{R}$.
  (b) Let $X = \mathbb{R}$ with co-countable topology, then every continuous function $f \colon X \to \mathbb{R}$ is constant.

(4) Define locally compact topological space. Does continous function preserves locally compactness ? Justify.

(5) Show that if $\widetilde{X}$ is the quotient of $\mathbb{R}^n (n \in \mathbb{N})$ obtained by the equivalence relation $a \sim b$ if and only if $a - b \in \mathbb{Z}^n$ for all $a, b \in \mathbb{R}^n$, then
    $\widetilde{X}$ is homeomorphic to n - dimensional torus $(\mathbb{S}^1)^n$.